Dust-Ion-Acoustic Solitary Waves in unmagnetized four components Dusty Plasma with Vortex-Like Electron distribution

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Abstract— A rigorous theoretical investigation has been made on four components dust-ion-acoustic (DIA) solitary waves (SWs) consisting Maxwellians negative ions, vortex-like electrons, arbitrarily charged stationary dust particles and cold mobile inertial positive ions in an unmagnetized dusty electronegative plasma (DENP) system. The properties of small but finite amplitude DIASWs are studied by employing the reductive perturbation technique. It has been found that owing to the departure from the Maxwellian electron distribution to a vortex-like one, the dynamics of such DIASWs is governed by a modified Korteweg-de Vries (mK-dV) equation which admits solitary wave solution under certain conditions. The basic properties (speed, amplitude, width, etc.) of such DIASWs, which are found to be significantly modified by the effects of trapped electrons and arbitrarily charged stationary dust particles. The implications of our results to space and laboratory dusty electronegative plasmas are briefly discussed.

Index Terms— Plasma, Dusty Plasma, Dust Ion Acoustic Waves, Solitary Waves, Electronegative Plasma, Maxwellians Distribution, Vortex-like Distribution, etc

1 INTRODUCTION

THE physics of charged dust particles, which are ubiquitous in space [1-4] and laboratory [4-7] plasmas has received a great deal of interest in understanding the electrostatic density perturbations and potential structures that are observed in space environments and laboratory devices. The work of Shukla and Silin [8] have theoretically shown that due to the conservation of equilibrium charge density $n_{i0} = en_{e0} + Z_d n_{do} e$, and the strong inequality [where n_{s0} is the equilibrium particle number density of the species s with $s = e(i)d$ for electrons (ions) dust, Z_d is the number of electrons residing onto the dust grains surface, and e is the magnitude of an electron charge], a dusty plasma system supports the low frequency (in comparison with the ion plasma frequency) dust-ion-acoustic (DIA) waves. The phase speed of such DIA waves is much smaller than the electron thermal speed but much larger than the ion thermal speed. The dispersion relation of the linear DIA waves is [8] $\omega^2 = n_{i0} k^2 C_i^2 / n_{e0}(1 + k^2 \lambda_{De}^2)$, where $\lambda_{De} = (k_B T_e / 4\pi n_{e0} e^2)^{1/2}$ is the electron Debye radius and $C_i = (k_B T_e / m_i)^{1/2}$ is the ion-acoustic speed with T_e being the electron temperature, m_i being the mass of ion, and k_B being the Boltzmann constant. At long wavelength limit (viz. $k^2 / \lambda_{De}^2 \langle\langle 1$), the dispersion relation for the DIA waves becomes $\omega = (n_{i0} / n_{e0})^{1/2} kC_i$. Therefore, this relation of DIA waves is similar to the usual ion-acoustic (IA) waves [9-11] for plasma with $n_{i0} = n_{e0}$ and $T_i \langle\langle T_e$ (where T_i is the ion temperature). But in dusty plasmas we usually have $n_{i0} \rangle\rangle n_{e0}$ and $T_i \approx T_e$. For this reason, dusty plasma cannot support the usual IA waves but it can support the DIA waves of Shukla and Silin [1]. The nonlinear propagation of the DIA waves in the dusty plasma have been studied by the several numbers of authors [12-14], but the DIA solitary waves in the electronegative plasma system is also the important field of attention. The plasma system which consisting a significant amount of negative ions [15-18] are called electronegative plasmas. The contribution of such negative ions in an plasma system cannot be neglected because of their potential applications in micro-electronic, photo-electronic industries [19], and the occurrence of these plasma system in both laboratory devices and space environments [16-22]. Electronegative plasmas are also known as dusty electronegative plasmas (DENP) [23-30] when dust particles are added to these plasmas. Actually, dust particles are not practically neutral but they are charged, i.e., it may be either positively or negatively charged [31] by absorbing electrons, positive ions as well as negative ions [23-30]. Therefore, a DENP is simply defined as normal electron-ion plasma with an additional charged component of micron/sub-micron sized dust particles. This additional component makes the plasma system very complex. This is why, a dusty plasma is also referred to as a ‘‘complex plasma’’ [32]. Recently, motivated by the laboratory experiment [33] Mamun *et al.* [34-36] have considered a dusty electronegative plasma containing Maxwellian electrons, Maxwellian negative ions, cold mobile positive ions, and negatively charged stationary dust, and have examined the possibility for the formation of IA and DIASWs and double layers in a DENP. Very recently, Rahman and Mamun [37] have studied the nonlinear propagation of DIASWs in an unmagnetized three

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component dusty plasma consisting of trapped electrons, cold mobile ions, and arbitrarily charged stationary dust. However, in our present work, we consider an unmagnetized four component DENP system containing trapped electrons, Maxwellian negative ions, cold mobile positive ions, and arbitrarily static dust positive ions, and arbitrarily charged stationary dust, and study the basic properties of the DIASWs.

The manuscript is organized as follows. The governing equations are presented in section II. The mK-dV equation is derived by employing the reductive perturbation method in section III. The solitary wave solution of this mK-dV equation is obtained and the properties of these DIA solitary structures are discussed in section IV. The numerical analysis of this work is presented in section V. A brief discussion is, finally, presented in section VI.

2 Governing Equations

We consider an unmagnetized four component collisionless DENP containing vortex-like electrons, Maxwellian negative ions, cold mobile positive ions, and arbitrarily charged stationary dust. Thus, at equilibrium we have, $n_{p0}-n_{e0}-n_{n0}+jZ_d n_{do}=0$, where n_{p0}, n_{e0}, n_{n0} and n_{d0} are, respectively, positive ion, electron, negative ion, and dust number density at equilibrium, Z_d is the number of electrons residing onto the surface of a stationary dust, and $j=+1(-1)$ for positively (negatively) charged dust. We are interested in examining the nonlinear propagation of a low phase speed (in comparison with electron and negative ion thermal speeds), long wavelength (in comparison with $\lambda_{De}=(k_BT_e/4\pi n_{e0}e^2)^{1/2}$ with T_e being the electron temperature, k_B being the Boltzmann constant, and e being the magnitude of the electric charge) perturbation mode on the time scale of the DIA waves. The time scale of the DIA waves is much faster than the dust plasma period so that dust can be assumed stationary. The nonlinear dynamics of the low frequency electrostatic perturbation mode in such a DENP is described by

$$\frac{\partial n_p}{\partial t}+\frac{\partial}{\partial x}(n_p u_p)=0 \tag{1}$$

$$\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}=-\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=\mu_e n_e-\mu_p-\mu_n n_n-jn_d \tag{3}$$

Where n_p (n_n) is the positive (negative) ion number density normalized by n_{p0} (n_{n0}), up is the positive ion fluid speed normalized by the positive ion-acoustic speed $C_p=(k_BT_e/m_p)^{1/2}$, ϕ is the electrostatic wave potential normalized by k_BT_e/e, $\mu_n=(1-\mu_e+j\mu_d)$, $\mu_d=Z_d n_{d0}/n_{p0}$, and m_p is the mass of positive ion. The time variable t is normalized by the positive ion plasma period ω_{pp}^{-1} and the space variable is normalized by λ_{De}.

The inertialess electron have a non-Maxwellians character, describe by a vortex-like velocity distribution which leads to a electron number density of the form [38-43],

$$n_e=1+\phi-\frac{4}{3}\frac{(1-\beta_e)}{\sqrt{\pi}}\phi^{3/2}+\frac{1}{2}\phi^2 \tag{4}$$

Where $\alpha=Z_nT_e/T_n$, $T_e\,(T_n)$ is the electron (negative ion) temperature, and β_e is a parameter which determine the number of trapped electrons and it is define by the ratio of the free electron temperature T_{ef} to the trapped electron temperature T_{et} [44, 45]. We note that $\beta_e=1$ $(\beta_e=0)$ represents a Maxwellian (flat-topped) electron distribution, whereas $\beta_e\langle 0$ represents a vortex-like excavated trapped electron distribution. We are interested in $\beta_e\langle 0$. On the other hand, the inertialess negative ion have a Maxwellians character describe by a Maxwellians Boltzmann velocity distribution which leads to a negative ion number density of the form

$$n_n=1+\alpha\phi+\frac{1}{2}\alpha^2\phi^2 \tag{5}$$

3 Derivation of MK-DV Equation

To study the electrostatic DIASWs in the DENP model under consideration, we construct a weakly nonlinear theory of the DIA waves with small but finite amplitude, which leads to a scaling of the independent variables through the stretched coordinates [37, 38, 40, 46, 47]

$$\left.\begin{aligned}\xi&=\varepsilon^{\frac{1}{4}}(x-v_pt)\\ \tau&=\varepsilon^{\frac{3}{4}}t\end{aligned}\right\} \tag{6}$$

where ε is a small parameter measuring the weakness of the dispersion, v_p is the unknown phase speed normalized by the positive IA speed C_p.

We can expand the perturbed quantities n_p, u_p and ϕ as [37, 46-49]

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$$\left.\begin{aligned} n_P &= 1+\varepsilon n_P^{(1)}+\varepsilon^{\frac{3}{2}}n_P^{(2)}+\ldots\ldots \\ u_P &= 0+\varepsilon u_P^{(1)}+\varepsilon^{\frac{3}{2}}u_P^{(2)}+\ldots\ldots \\ \phi &= 0+\varepsilon\phi^{(1)}+\varepsilon^{\frac{3}{2}}\phi^{(2)}+\ldots\ldots \end{aligned}\right\} \tag{7}$$

Now, using (4)-(7) into (1)-(3) one can obtain the first order continuity equation, momentum equation, and Poisson's equation which, after simplification, yield

$$n_p^{(1)} = \frac{\phi^{(1)}}{v_p{}^2} \tag{8}$$

$$u_p^{(1)} = \frac{\phi^{(1)}}{v_p} \tag{9}$$

$$v_p = \frac{1}{\sqrt{(1+j\mu_d)-(1-\alpha)\mu_n}} \tag{10}$$

Equation (10) represents the linear dispersion relation for DIA waves. Putting the values of eqs (4)-(10) into eqs (1)-(3), we obtain the next higher order equations,

$$\frac{\partial n_p^{(1)}}{\partial\tau} - v_p\frac{\partial n_p^{(2)}}{\partial\xi} + \frac{\partial u_p^{(2)}}{\partial\xi} = 0 \tag{11}$$

$$\frac{\partial u_p^{(1)}}{\partial\tau} - v_p\frac{\partial u_p^{(2)}}{\partial\xi} + \frac{\partial\phi^{(2)}}{\partial\xi} = 0 \tag{12}$$

$$\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = \frac{\phi^{(2)}}{v_p^2} - \frac{4(1-\beta_e)}{3\sqrt{\pi}}\mu_e[\phi^{(1)}]^{\frac{3}{2}} - n_p^{(2)} \tag{13}$$

Now, using eqs. (11)-(13) one can easily eliminate ($\frac{\partial n_p^{(2)}}{\partial\xi}$), ($\frac{\partial u_p^{(2)}}{\partial\xi}$) and ($\frac{\partial\phi^{(2)}}{\partial\xi}$) and obtain

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\sqrt{\phi^{(1)}}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0 \tag{14}$$

Where,

$$A = \frac{(1-\beta_e)\mu_e v_p^3}{\sqrt{\pi}} \tag{15}$$

$$B = \frac{v_p^3}{2} \tag{16}$$

Equation (14) is a modified Korteweg-de Vries (mK-dV) equation, exhibiting a stronger nonlinearity, smaller width, and larger propagation velocity of the nonlinear wave.

4 SW Solution of MK-Dv Equation

The stationary solution of this mK-dV equation can be obtained by transforming the independent variables ξ and τ to $X=\xi-u_0\tau$, $\tau=\tau$, where u_0 is a constant velocity. Now using the appropriate boundary conditions for localized disturbances,

viz. $\phi^{(1)}\to 0$, $(d\phi^{(1)}/dX)\to 0$, $(d^2\phi^{(1)}/dX^2)\to 0$, at $X\to\pm\infty$. Thus, one can express the stationary solution of this mK-dV equation as

$$\phi^{(1)} = \phi_m \sec h^4[(\xi-u_0\tau)/\Delta] \tag{17}$$

Where the amplitude ϕ_m and width Δ are given by $\phi_m=(15u_0/8A)^2$ and $\Delta=\sqrt{16B/u_0}$, respectively

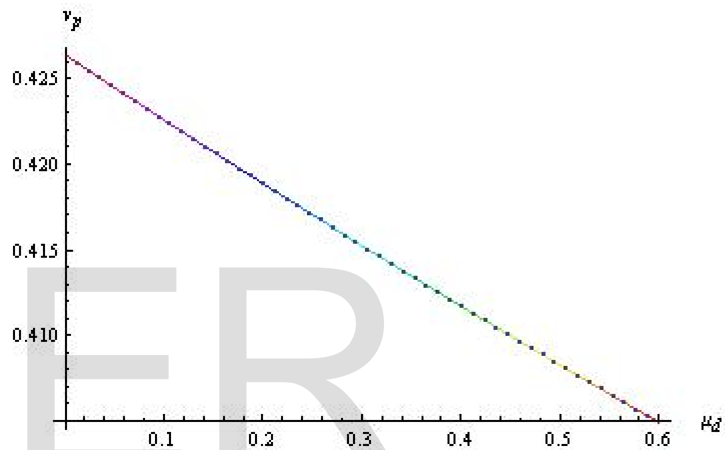

Figure 1: Showing the variation of the phase speed (v_p) of solitary waves with μ_d for *j=+1*, μ_n =0.5 and α *=10*.

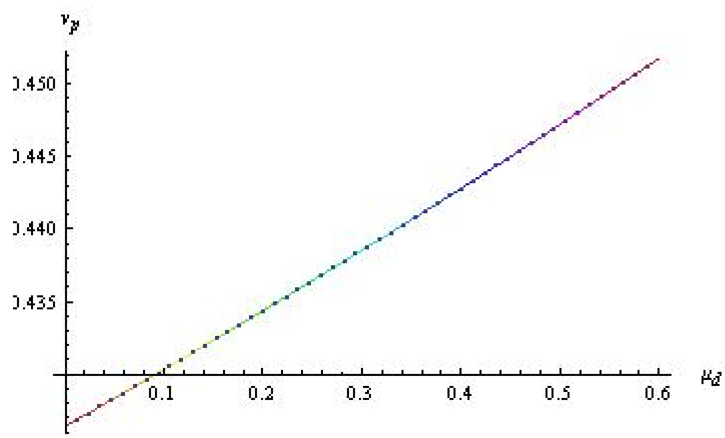

Figure 2: Showing the variation of the phase speed (v_p) of solitary waves with μ_d for *j=-1*, μ_n =0.5 and α *=10*.

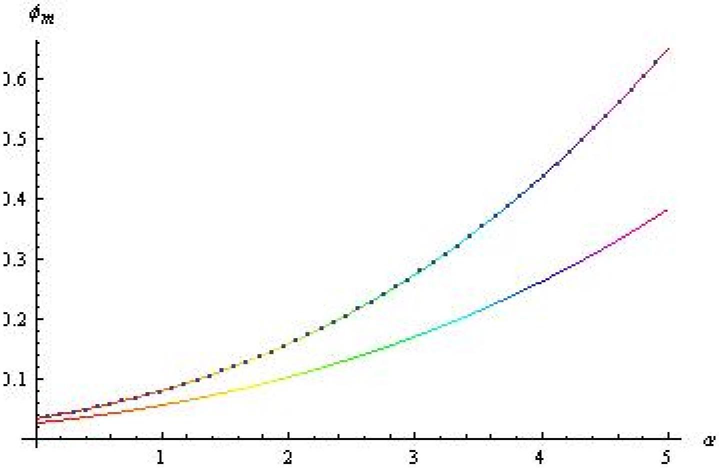

Figure 3: Showing the variation of the amplitude (ϕ_m) of solitary waves with α for *j=±1, u_0=0.1, μ_n=0.3, μ_d=0.2,* and β_e=−0.7.

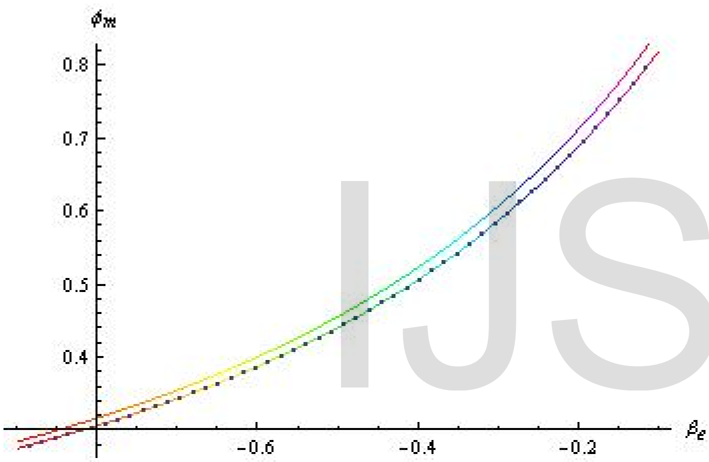

Figure 4 Showing the variation of the amplitude (ϕ_m) of solitary waves with β_e for *j=±1, u_0=0.1, μ_n=0.2, μ_d=0.02, and* α=5.

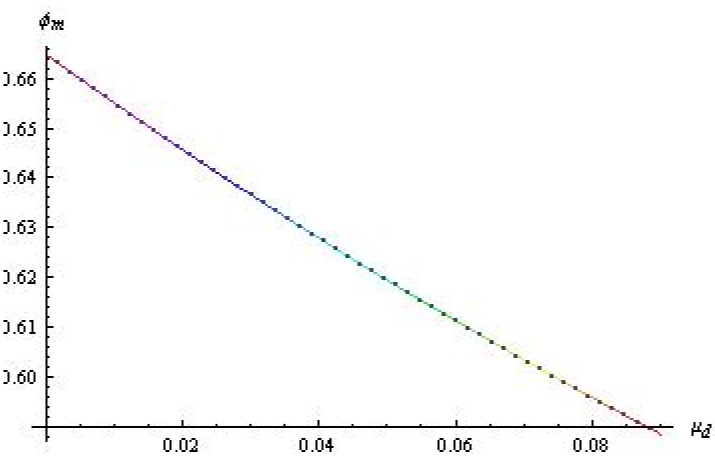

Figure 5: Showing the variation of the amplitude (ϕ_m) of solitary waves with μ_d for *j=+1, u_0=0.1, μ_n=0.3, β_e=-0.9, and* α=5.

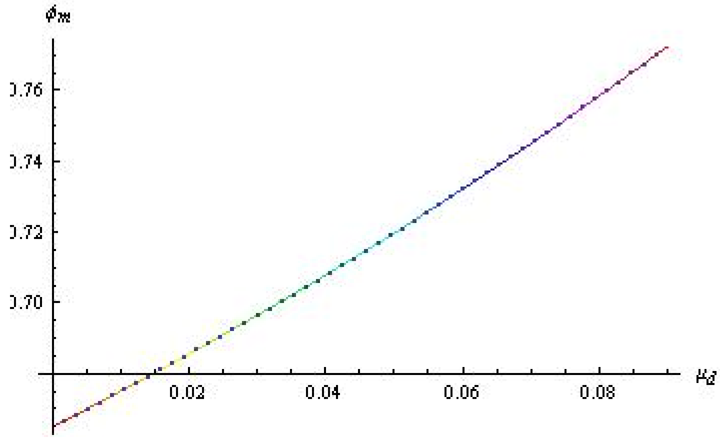

Figure 6: Showing the variation of the amplitude (ϕ_m) of solitary waves with μ_d for *j=-1, u_0=0.1, μ_n=0.3, β_e=-0.9, and* α=5.

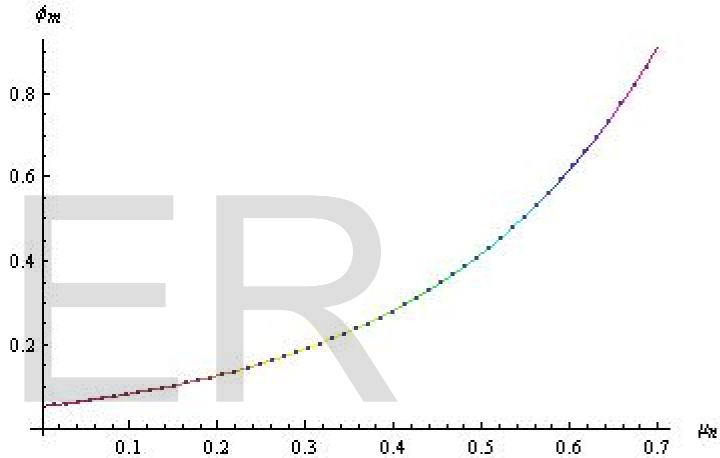

Figure 7: Showing the variation of the amplitude (ϕ_m) of solitary waves with μ_n for *j=+1, u_0=0.1, μ_d=0.7, β_e=-0.9, and* α*=3.*

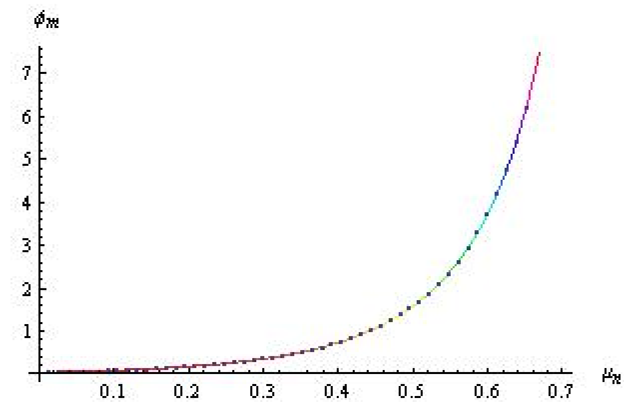

Figure 8: Showing the variation of the amplitude (ϕ_m) of solitary waves with μ_n for *j=-1, u_0=0.1, μ_d=0.09, β_e=-0.7, and* α*=3.*

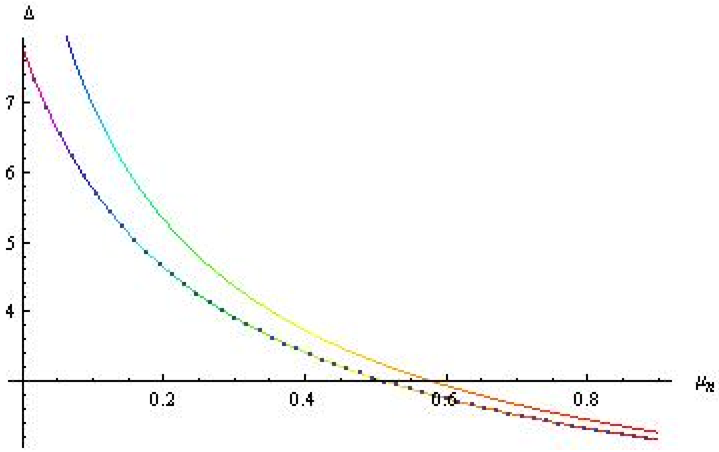

Figure 9: Showing the variation of the width (Δ) of solitary waves with μ_n for *j=±1, u_0 =0.1, μ_d =0.2, and α =7.*

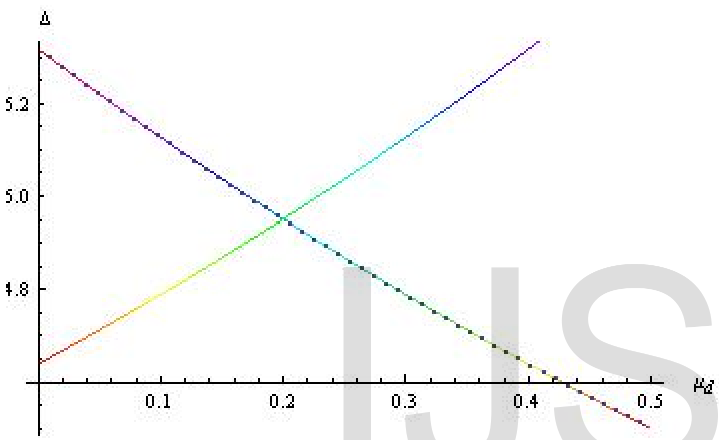

Figure 10: Showing the variation of the width (Δ) of solitary waves with μ_d for *j=+1(-1), u_0 =0.1, μ_n =0.2, and α =6(7).*

5 Numerical Analysis

Figure 1 shows the variation of the phase speed v_p of solitary wave with μ_d for α =10, μ_n =0.5, and *j=+1*. Figure 2 shows the variation of the phase speed v_p of solitary wave with μ_d for α =10, μ_n =0.5, and *j=-1*. From figure 1, it has been found that the phase speed of the solitary wave decreases with μ_d in case of positively charged dust but it increases with μ_d for negatively charged dust which is shown in figure 2. Figure 3 shows the variation of the amplitude of solitary wave potential ϕ_m with α for u_0 =0.1, μ_n *=0.3,* μ_d *=0.2,* β_e =-0.7, and j=±*1*. In this figure, the dotted curve shows the variation of ϕ_m with α for positively charged dust and solid curve for negatively charged dust. This figure indicates that the amplitude increases with increasing the value of α either dust particle is positive or negative but the amplitude is higher for positively charged dust than for negatively charged dust. Figure 4 shows the variation of the amplitude of solitary wave potential ϕ_m with β_e for u_0 =0.1, μ_n =0.2, μ_d =0.02, α =5, and *j=±1*. In this figure, the dotted curve shows the variation of ϕ_m with β_e for positively charged dust and solid curve for negatively charged dust. This figure also indicates that the amplitude increases with increasing the value of β_e either dust particle is positive or negative but the amplitude is slightly larger for negatively charged dust than for positively charged dust. Figure 5 shows the variation of the amplitude of solitary wave potential ϕ_m with μ_d for u_0 =0.1, α =5, β_e =-0.9, μ_n =0.5, and *j=+1* . Figure 6 shows the variation of the amplitude of solitary wave potential ϕ_m with μ_d for u_0 =0.1, α =5, β_e =-0.9, μ_n =0.5, and *j=-1*. From figure 5 it has been found that the amplitude of the solitary wave decreases linearly with μ_d in case of positively charged dust but it increases linearly with μ_d for negatively charged dust which is shown in figure 6. Figure 7 shows the variation of the amplitude of solitary wave potential ϕ_m with μ_n for u_0 =0.1, μ_d =0.7, α =3, β_e =-0.9, and *j=+1*. From figure 7, it has been seen that the amplitude increases with increasing the value of μ_n . Figure 8 shows the variation of the amplitude of solitary wave potential ϕ_m with μ_n for u_0 =0.1, μ_d =0.09, α =3, β_e =-0.7, and *j=-1*. From figure 8, it has been seen that the amplitude increases with increasing the value of μ_n and in that case amplitude is very large compared with positively charged dust. Figure 9 shows the variation of the width Δ of solitary wave with μ_n for u_0 =0.1, μ_d =0.2, α =7, and *j=±1*. In this figure, the dotted curve shows the variation of Δ with μ_n for positively charged dust and solid curve for negatively charged dust. This figure indicates that the width decreases with increasing the value of μ_n either dust particle is positive or negative but the width is slightly lower for positively charged dust than for negatively charged dust. Figure 10 shows the variation of the width Δ of solitary wave with μ_d for u_0 *=0.1,* μ_n =0.2, α =6(7), and *j=+1(-1)*. This figure shows how the width Δ of solitary wave varies with μ_d for positively (negatively) charged dust dotted curve (solid curve). From this figure it is clear that the width decreases with increasing the value of μ_d for positively charged dust but it increases for negatively charged dust.

6 Discussion

We have considered an unmagnetized four component DENP system consisting of electrons which obeying vortex-like distribution, negative ions satisfying Maxwellian distribution, cold mobile positive ions, and arbitrarily charged stationary dust, and have studied the basic properties of DIASWs by deriving mK-dV equation. It has been observed that the effect of dissipation in this case is negligible in comparison with those of the nonlinearity

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and dispersion. The basic features (phase speed, amplitude, and width) of such DIA solitary waves are significantly modified by the presence of trapped electron, and arbitrarily charged stationary dust particles. The results which have been obtained from this investigation can be summarized as follows:

I. Dusty plasma system, whose constituents are cold mobile positive ions, trapped electrons, Maxwellian negative ions, and arbitrarily charged stationary dust of different constant temperatures, are found to support solitary waves associated with the non-linear DIA waves.

II. In the presence of the trapped electron distribution, the dynamics of weakly dispersive non-linear DIA waves is governed by the mK-dV equation, the stationary solution of which is represented in the form of an inverted secant hyperbolic fourth profile. Thus, the potential polarity of the DIA solitary waves in our dusty plasma is different from the usual IA solitary waves in an electron-ion plasma.

III. The profile of this wave can be represented in the form of $\operatorname{sec}h^4(X/\Delta)$, instead of $\operatorname{sec}h^2(X/\Delta)$ which is the stationary solution of the standard K-dV equation for isothermal electrons.

IV. The dusty plasma system under consideration supports the DIA solitary waves that are associated with positive potential only. The fixed polarity of the potential structures is due to the effect of trapped electron distribution.

V. It has been found that the effect of trapped electron gives rise to a stronger nonlinearity (i.e., we have solitary structures with larger amplitude, smaller width, and higher propagation velocity).

VI. From these above figures (1-10), it has been also found that the phase speed of the solitary wave decreases with increasing the values of μ_d if dust particles are positive on the other hand it increases with μ_d if dust particles are negative. The amplitude of the solitary wave increases with increasing the values of α and β_e either dust particles are positive or negative. Whereas, the width of the solitary waves decreases with increasing the value of μ_n either dust particles are positive or negative and it increases with μ_d when dust particles are negative.

We hope that our present results should help to modify to understand the basic features of the electrostatic disturbances in space and laboratory devices. The present work can also provide a guideline for interpreting the most recent numerical simulation results, which exhibit the simultaneous presence of non-thermal ion distributions and associated DIA localized wave packets.

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